Design Factors of the Bell Telephone Laboratories 1553 Triode

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(*Manuscript Received Aug. 3, 1950*)

IN DEVELOPING microwave relay systems for frequencies around 4000 megacycles, one of the major problems is to provide an amplifier tube which will meet the requirements on gain, power output, and distortion over very wide bands. As the number of repeaters is increased to extend the relay to greater distances, the requirements on individual amplifiers for the system become increasingly severe. A tube developed for this service is the microwave triode B.T.L. 1553, the physical and electrical characteristics of which were briefly described in a previous article.[1] In the development of such a tube, both theoretical and experimental factors are involved; illustration of these factors in some detail is the purpose of the present paper.

Given the application, a number of questions arise at the outset. What determines the tube type—why pick a triode for development, rather than a velocity variation tube, or perhaps a tetrode? What electrode spacings are necessary in such a tube, and what current must it draw? How is its performance rated, and how does it compare with other tubes? To what extent can the performance be estimated in advance? What experimental tests can give more precise information? Some answers to these questions were obtained by the use of figures of merit, which led up to the choice of a triode as most promising for development, and which also led to the subsequent method of optimizing the design for the particular system application of microwave amplifiers and modulators.

The design process may be said to proceed by the following series of steps:

1. Formulate the system requirements, frequently with the aid of one or more figures of merit. The purpose here is to concentrate attention upon the limitations inherent in the tube alone by eliminating considerations of circuitry or of other parts of the system. The figure of merit measures tube performance in an arbitrary environment, so chosen as to be simple, and also directly comparable to the actual system requirement.
2. Make tentative choices of tube type, and analyze further to find out

[1] J. A. Morton, "A Microwave Triode for Radio Relay," *Bell Laboratories Record* 27, 166–170 (May 1949).

how the figure of merit depends on the internal parameters of the tube, such as spacings, current density, and so on.

3. Optimize the internal parameters to make the figure of merit as good as possible, with due regard to practical limitations like cathode activity, life, cost, etc.

4. Use enough experimental checks to make sure the estimates are sound. Build then the type of tube which appears to fill the requirements best, including the practical as well as the technical limitations. The figures of merit serve now as quantitative checks both of how well the tube satisfies the application, and also of how accurate is the theory.

Given a good accurate design theory, the whole process could in principle be calculated in advance. Such a theory would permit great savings in effort, since spot checks of relatively few parameters are sufficient to insure accuracy even when the theory is used to predict a wide range of phenomena. The extent to which presently available microwave tube theory meets this need is considerable, as will appear from some of the results below.

The degree of accuracy required of a theory increases as the development process continues. For preliminary estimates, such as deciding what tube type to develop, the theory can be rather rough and still be satisfactory. For complete predictions of final performance, only experimental construction can suffice. By this means the theory can be checked, so that it can serve future designs with improved accuracy.

The method outlined here is not new, but rather follows standard practice fairly closely. It does, however, give more than usual quantitative emphasis to the figures of merit, using them to codify the procedure; and it incorporates a certain amount of quantitative calculation at microwave frequencies. It will be seen that the theory of Llewellyn and Peterson needs only some semi-empirical supplementation in the low-voltage input space, as has already been pointed out by Peterson.[2]

Preliminary Estimates—Choice of Tube Type

For the New York to Boston microwave relay, an output amplifier was developed using already available velocity-modulation tubes.[3] With four stagger-tuned stages, the amplifier proved satisfactory for this service, and in fact tests indicated that this system could be extended to considerably greater distances and still give good performance. It was apparent, however, that these amplifiers would not be satisfactory for a coast-to-coast system.

[2] L. C. Peterson, "Signal and Noise in Microwave Tetrodes," *I. R. E. Proc.* (Nov. 1947).

[3] H. T. Friis, "Microwave Repeater Research," *B. S. T. J.* 27, 183–246 (April 1948).

When this limitation became clear several years ago, a study was undertaken to determine which particular type of electron tube amplifier then known had the best possibilities of being pushed to greater gain-band products. The results of this study indicated that a very promising prospect was to build, for operation at 4000 megacycles, an improved planar triode, that is, one in which the active elements are on parallel planes.

In arriving at this conclusion, two general types of device were considered: velocity-modulated, as in a klystron, and current-modulated, as in a triode. (Nowadays, such a study would of course include traveling-wave tubes.) The conclusions were reached with the aid of the gain-band figures of merit, along the following lines:

Gain-Band Product

The system performance requirements demand amplifiers capable of reasonable gains and power outputs over prescribed bandwidths. However, it is known that bandwidth can be increased by complicating the circuits (double-tuning, stagger-tuning, etc.). Such factors, being common to whatever tube may be used, are extraneous to a discussion of tube performance, and accordingly the tubes are rated by their performance with simple, synchronous resonant circuits. Furthermore, even then the bandwidth can be increased at the expense of a corresponding reduction of gain, by simply depressing the impedance levels of the interstages. Since the product of gain and bandwidth remains constant, it is a suitable figure of merit, independent of the particular choice of bandwidth, provided the definition of gain is suited to the device.

Unfortunately there is more than one possible gain-band product, the appropriate form depending on how many simple resonant circuits shape the band of the amplifier stage. For example, a conventional pentode or a velocity-variation tube is usually used in conjunction with two high-Q resonant circuits, one each on input and output. If these are adjusted to give the same Q, then it is well known that, no matter what the bandwidth, the product of voltage gain and bandwidth is constant. (See Appendix 1)

$$|\Gamma_0|B = |Y_{21}|/2\pi\sqrt{C_{\text{in}}C_{\text{out}}} \tag{1}$$

Here Γ_0 is the mid-band voltage gain, B the bandwidth 6 db down (3 for each circuit), Y_{21} the stage transadmittance, and C_{in} and C_{out} the total effective capacitances of the resonant circuits, including the contributions of the tube*. It is assumed that the stage is matched into transmission lines of some suitable constant admittance level G_0.

In amplifiers using triodes such as the B.T.L. 1553 (or tetrodes) in

* As shown in Appendix 1, all quantities in equations (1) and (2) are the values effective at the electrodes adjacent to the electron stream.

grounded-grid circuits, the situation is different because the Q of the input circuit is always very much smaller than that of the output. Here a figure of merit independent of bandwidth is obtained from the product of power gain and bandwidth:

$$|\Gamma_0|^2 B = |Y_{21}|^2/4\pi G_{in} C_{out} \tag{2}$$

Here G_{in} is the total conductance of the input circuit, including tube contributions*. The gain is again measured with the tube matched at an arbitrary admittance level G_0. The band, being now limited by only one tuned circuit, is somewhat different in shape from the above, and is taken 3 db down.

While each figure of merit gives an unequivocal rating of tubes of appropriate type, the intercomparison of the two types still depends on the bandwidth. In particular, as the band is widened, the two-circuit type (klystron) loses gain at the rate of 6 db per octave of bandwidth, while the one-circuit type (triode) loses only 3 db per octave. Consequently, if the two devices start with equal gains at some narrow bandwidth, the triode rapidly pulls ahead in gain as the bandwidth is increased.

The figure of merit equation (1) states that improved klystron performance implies either an increase in transadmittance Y_{21} or a decrease in the band-limiting capacitances C_{in} or C_{out}. According to the simplest klystron bunching concept,[4] the transconductance of such a tube may be increased indefinitely simply by making the drift time longer. Unfortunately, this simple kinetic picture does not take account of the mutually repulsive space-charge effects which set an upper limit to the useful drift time by debunching the electrons.[5] For a 2000-volt beam in the 4000-megacycle range, this limit is approximately three micromhos per milliampere. The 402A tube used in the New York to Boston system has already approached this limit within a factor of two. Since the capacitances are also quite small, the prospect is quite dubious for any considerable improvement in gain-band merit if the simple klystron type of operation were to be used.

Improvements are possible in a klystron by changing the manner of operation so as to lower the drift voltage V_0, because the aforesaid transadmittance limit is proportional to V_0^{-n} .* This prospect is also relatively unattractive. To get transadmittance values anywhere near the triode would require low voltages and close spacings somewhat like the latter, and would encounter space-charge difficulties involved in handling a large current in a low-voltage drift space. Furthermore, the tube would be more

[4] D. L. Webster, *Jour. App. Phys.* 10, 501–508 (July 1939).
[5] S. Ramo, *Proc. I. R. E.* 27, 757–763 (December 1939).
* The value of n may vary between 1/4 and 3/4. See reference 5.

complex, having several grids instead of one. A number of modifications of klystron operation were considered, but all looked more complex mechanically and more speculative theoretically than a triode.

In a triode there is also an upper limit to the transconductance that can be achieved by spacing cathode and grid more closely. This limit would be reached if the spacing were so close that the velocity produced by the grid voltage were of the same order as the average thermal velocity of cathode emission. The triode limit of some 11,000 micromhos per milliampere is, however, many times greater than that for ordinary klystrons. What is still more important is the fact that previous microwave triodes were still a factor of twenty to twenty-five below this limit, leaving considerable room for improvement. Thus, if mechanical methods could be devised for decreasing the cathode-grid spacing and at the same time maintaining parallelism between cathode and grid, it seemed highly probable that great improvements would be available from a new triode.

The choice to develop a triode for this application was therefore taken not merely on the basis of simplicity, but also with the expectation that performance improvements would be not only larger but also more certainly obtainable than by use of a modified klystron. Moreover, the possibilities of using the triode over a wide frequency range in other ways—as a low noise amplifier, modulator and oscillator—lent additional weight to its choice. By translating the known requirements on gain, bandwidth and power output into triode dimensions as discussed below, it was found that the input spacings of existing commercial tubes would have to be reduced by a factor of about five. In addition, cathode emission current densities would have to be increased about three times. A design was evolved in which the required close spacings could be produced to close tolerances by methods consistent with quantity production requirements. The B.T.L. 1553 tube was the result (Fig. 1). Many of its design features were adopted for use in the Western Electric 416A tube, which is an outgrowth of this investigation.

Description of B.T.L. 1553 Triode*

The electrode spacings of this tube and of a 2C40 microwave triode are shown in Fig. 2. In the 1553, the cathode-oxide coating is .0005″ thick, the cathode grid spacing is .0006″, the grid wires are .0003″ in diameter, wound at 1000 turns per inch, and the plate-grid spacing is .012″. It is interesting to note that the whole input region of the 1553 including the grid is well within the coating thickness of the older triode.

The arrangement of the major active elements of the tube is shown in

* This section is repeated from reference 1 for completeness.

Fig. 3. This perspective sketch has been made much out of scale so that the very close spacings and small parts would be seen. The nickel core of the cathode is mounted in a ring of low-loss ceramic in such a manner that the nickel and ceramic surfaces may be precision ground flat and coplanar. A thin, smooth oxide coating is applied to the upper surface of

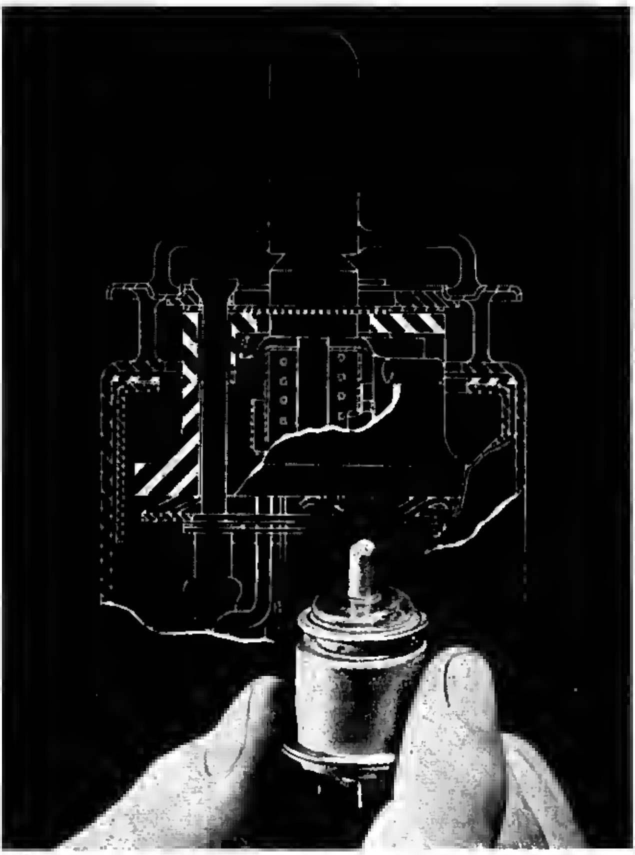

Fig. 1.—The B.T.L. 1553 microwave triode with a cross-section drawing of it in the background.

the cathode by an automatic spray machine developed especially for this tube. With this machine, a coating of $0.0005'' \pm 0.00002''$ may be put on under controlled and specifiable conditions. To insure long life with such a thin coating, it was necessary to develop coatings from two to four times as dense as those used in existing commercial practice.

The grid wires are wound around a flat, polished molybdenum frame

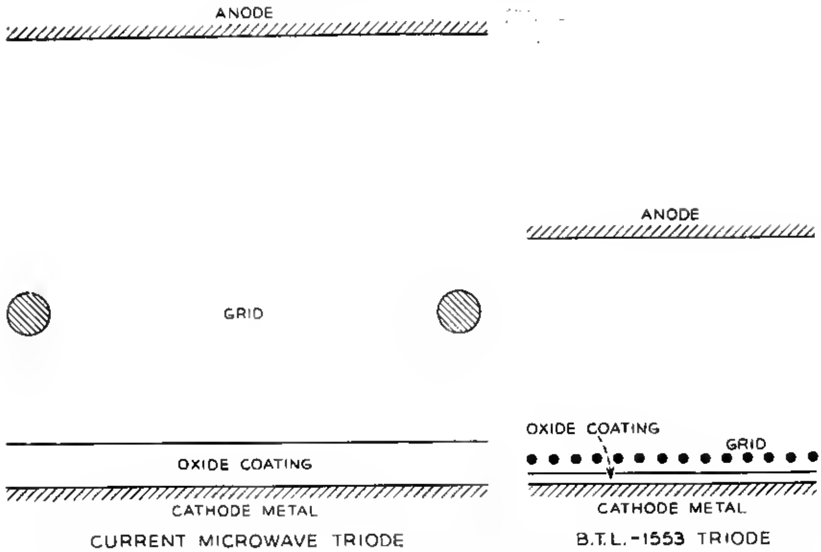

Fig. 2.—Comparison of the spacings of the 1553 triode at the right with a previously existing microwave triode at the left.

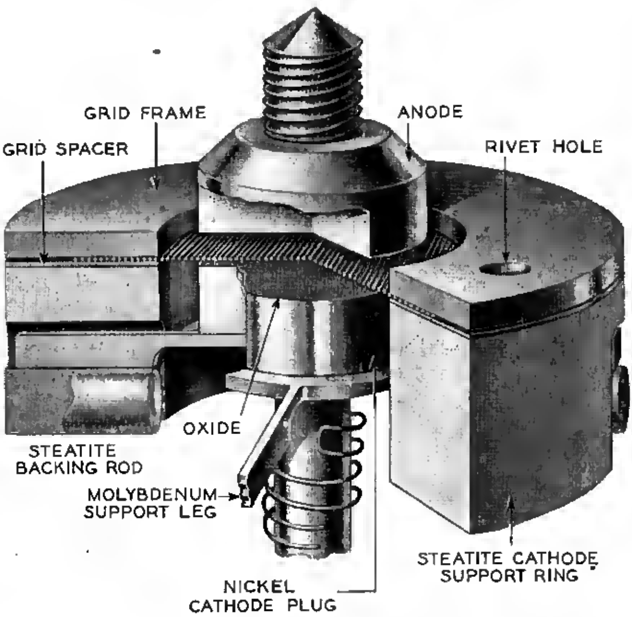

Fig. 3.—Perspective drawing of the active elements of the 1553 close-spaced triode.

that has been previously gold sputtered. The winding tension is held within ± 1 gram weight to about 15 gram weight, which is about sixty per cent of the breaking strength of the wire. This is accomplished by means of a small drag-cup motor brake, a new method which was developed especially for these fine grids. The grid is then heated in hydrogen to about 1100°C, at which point the gold melts and brazes the wires to the frame. The mean deviation in wire spacing is less than about ten per

Fig. 4.—Physical appearance of the elements comprising the 1553 triode.

cent, and in fact these grids are fine enough and regular enough to be diffraction gratings as is shown in Fig. 5. In this figure, a fourth order spectrum diffracted by one of these grids can be seen. The third order, which should be absent because the wire size is about one-third of the pitch, is much less intense than the fourth. Proper spacing of the grid is then obtained by a thin copper shim placed between the cathode ceramic and the grid frame. Its thickness must be equal to the coating thickness, plus the thermal motion of the cathode, plus the desired hot spacing.

The cathode, spacer, and grid comprising the catbode-grid subassembly are riveted together under several pounds of force maintained by the molybdenum spring on tbe bottom of the assembly. The rivets are three synthetic sapphire rods fired on the ends with matcbing glass. In Fig. 4, the parts comprising this assembly are sbown in appropriate pile-up sequence at the left, and the completed catbode-grid subassembly is shown at the right between the bulb and tbe press. Tbe grid-anode spacing of .012″ is easily obtained by means of an adjustable anode plug the surface of wbich is gauged relative to the bulb grid disc.

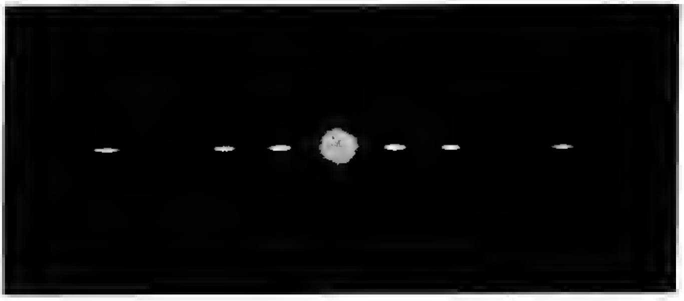

Fig. 5.—Spectrum formed by the grid of the 1553 microwave triode.

TABLE I
LOW-FREQUENCY CHARACTERISTICS
For $V_p = 250\ V$, $I_p = 25$ ma, $V_g = -0.3\ V$

$g_m = 50{,}000\ \mu$mhos $\mu = 350$ $r_p = 7000$ ohms	$C_{kg} = 10\ \mu\mu f$ $C_{gp} = 1.05\ \mu\mu f$ $C_{kp} = .005\ \mu\mu f$

The higher current density of 180 milliamperes per square centimeter, the thin dense catbode coating, and the very close spacings, posed a problem in obtaining adequate emission and freedom from particle shorts, and had to be solved by quality control methods because of the large number of factors involved and the precision required. Tubes, subassemblies, and testers have been made in batches and studied by statistical methods. To achieve a state of statistical control on emission, and freedom from dust particles, it is necessary to process the parts and assemble tbe tubes in a rigorously controlled environment. Completely air-conditioned processing and assembly rooms operating under rigorous controls have been found necessary[6]. Under such controlled conditions, good production yields with satisfactory catbode activity have been obtained,

[6] R. L. Vance, *Bell Laboratories Record*, 27, 205–209 (June 1949).

whereas without such conditions not only was the yield low but it was difficult to ascertain just what factors were operating to inhibit emission and to cause cathode-grid shorts.

A summary of the pertinent low-frequency characteristics of the 1553 triode is given in Table I. It should be noticed that, at plate currents of 25 milliamperes, the transconductance per milliampere is about 2000, that is, about one-fifth of the theoretical upper limit. At lower currents this ratio is higher: at 10 milliamperes, for example, it is 3000 micromhos per milliampere. Diodes with the same spacings have about twice these values of transconductance per milliampere, showing that the grid is fine enough to obtain fifty per cent of the performance of an ideal grid.

Triode Design Requirements

Analysis of the figure of merit can well begin by devoting attention to the band-limiting capacitance C_{out} of the output circuit. First, some question may be raised as to the applicability of the concept of a simple L-C shunt resonant circuit at high frequencies, where the circuit parameters are actually distributed, not lumped. Suppose the actual circuit admittance is $Y_x = G_x + jB_x$. In order to represent it as a simple shunt resonant circuit of admittance $Y_p = G_p + j\omega C_p + 1/j\omega L_p$, we need only require that the two be equal and have equal derivatives with respect to frequency at the center frequency $f_0 = \omega_0/2\pi$. Accordingly the "effective values" of the actual admittance are given by the following equations:

$$\begin{aligned} G_p &= G_x(\omega_0) \\ C_p &= \tfrac{1}{2}(B_x' + B_x/\omega_0) \\ \frac{1}{L_p} &= \tfrac{1}{2}(\omega_0^2 B_x' - \omega_0 B_x) \end{aligned} \tag{3}$$

From this development one sees that the representation neglects G_x', the first derivative of the conductance, but otherwise is correct to first order as a function of frequency.

There are important cases where this representation as a simple circuit does not hold. For example, double-tuned circuits having two local resonances have a fundamentally different band sbape. However, such complication of the circuits has been excluded from the figure of merit on the ground that it is purely a circuit "broad-banding" problem: having determined the performance of the tube for simple circuits, any broad-banding (double-tuning, staggering, etc.) will give a calculable improvement which does not depend upon the tube. Accordingly, to compare tubes it is sufficient to consider standard simple circuit terminations, tuned to the same frequency.

The total capacitance C_{out} includes two contributions: from the active electrode area inside the tube (C_{22}) and from the passive resonating circuit (C_{p2}). It is convenient to consider these separately, writing the figure of merit as follows:

$$|\Gamma_0|^2 B = \frac{|Y_{21}^2|}{4\pi G_{11} C_{22}} \frac{1}{\left(1 + \frac{G_{p1}}{G_{11}}\right)\left(1 + \frac{C_{p2}}{C_{22}}\right)} \tag{4}$$

The first factor is the "intrinsic" electronic figure of merit of the active transducer alone, while the second factor expresses the deterioration caused by input passive circuit loss G_{p1} and output passive circuit capacitance C_{p2}, both of which should ideally be held as small as possible.

Consider the first factor, the intrinsic electronic gainband product which depends only upon the properties of the electron stream and the electrode dimensions in the regions occupied by the electron stream.

It is the responsibility of the tube design engineer to maximize this product consistent with any limitations which may be imposed by mechanical, emission, thermal or circuital considerations.

On the other hand, in maximizing this intrinsic gain-band product, the tube engineer must not proceed in ignorance of the effect of his actions on the possibility of obtaining a favorable value for the second factor. For example, he may attempt to make C_{22} so small (in order to maximize the first factor) that it becomes physically impossible to obtain an effective circuit capacitance C_{p2} which is not large compared to C_{22}. In such a case, the actual gain-band product would be much smaller than the intrinsic product of which the tube would be capable if circuit capacitance were negligible. Such a balancing of effects will become apparent from the subsequent discussion.

It is desired, therefore, to express the transadmittance, input conductance and output capacitance of the electronic transducer in terms of such parameters as cathode current density, electrode dimensions, frequency and potentials in such a way that it will become clear bow a maximizing process may be carried out by adjusting these parameters.

As a first approximation let us use the results of Llewellyn and Peterson's analysis of plane-parallel flow[7], which makes the following assumptions:

1. All electrons are emitted with zero velocity.
2. All electrons in a given plane have the same velocity.

[7] F. B. Llewellyn and L. C. Peterson, "Vacuum Tube Networks," *Proc. I. R. E.*, 32, 144–166 (1944).

3. The dimensions of the grid are infinitesimal compared to the electrode spacings.
4. The electrode dimensions are small compared to the wavelength.

It can be shown that the intrinsic gain-band product may be expressed in the following two ways:

$$
\begin{aligned}
M_i &= K\left[\frac{1}{x_1}\right]\left[\frac{F_1^2(\theta_1)}{\theta_1 F_3(\theta_1)}\right][\theta_2 F_2^2(\theta_2)\sqrt{V_p}] \\
&= K'\left[\frac{1}{j}\right]\left[\frac{F_1^2(\theta_1)}{\theta_1^4 F_3(\theta_1)}\right][\theta_2 F_2^2(\theta)\sqrt{V_p}]
\end{aligned}
\tag{5}
$$

where K, K' are parameters which are functions only of frequency.

x_1 is the cathode-grid spacing in cm

θ_1 is cathode-grid transit angle and $\theta_1 = \dfrac{126}{\lambda}\left(\dfrac{x_1}{j}\right)^{1/3}$

j = cathode current density in amp/cm^2

θ_2 = grid-anode transit angle and $\theta_2 = \dfrac{6300\, x_2}{\lambda\sqrt{V_p}}$

and $F_1(\theta_1)$, $F_2(\theta_2)$ and $F_3(\theta_1)$ are complicated functions of their respective transit angles.

Consider frequency to be given as part of the specifications on the tube.

Variation with Current Density, j

In the first formulation the current density is involved only in the second factor. This factor is a function only of $\theta_1 = \left(\dfrac{x_1}{j}\right)^{1/3}$ and is shown plotted in Fig. 6. If x_1 and λ are considered to be held fixed for the moment the first maximum at $\theta_1 \to 0$ requires j to be as large as possible consistent with emission limitations and life. For the 1553 the cathode current density is set at 180 ma/cm^2.

The other maxima at larger values of θ_1 (and smaller values of j), where $F_3(\theta_1)$ goes through zero, correspond to transit angles where $G_{11} \to 0$ in the single-valued velocity theory. These maxima cannot be taken at face value, however, to indicate maxima in the unequal-Q gain-band product since they violate the assumption that $Q_1 << Q_2$ for which the formula was developed. To make a study of gain-band variation in this region therefore entails a study of gain-band product as a function of bandwidth, as was pointed out previously in connection with comparison of the equal-Q and unequal-Q cases. Such maxima are of interest pri-

marily in narrow band cases so that for the present we shall concern ourselves only with the first maximum at $\theta_1 \to 0$ and j indefinitely large.

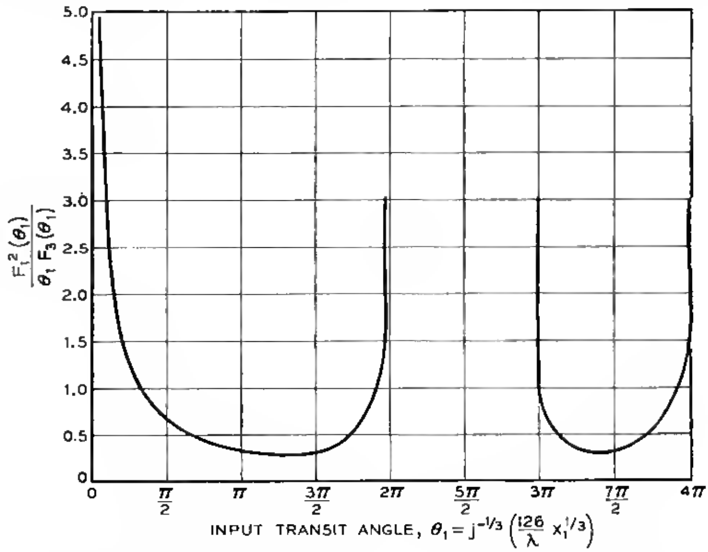

Fig. 6.—Gain-band product dependence on current density (j), with input spacing (x_1) fixed.

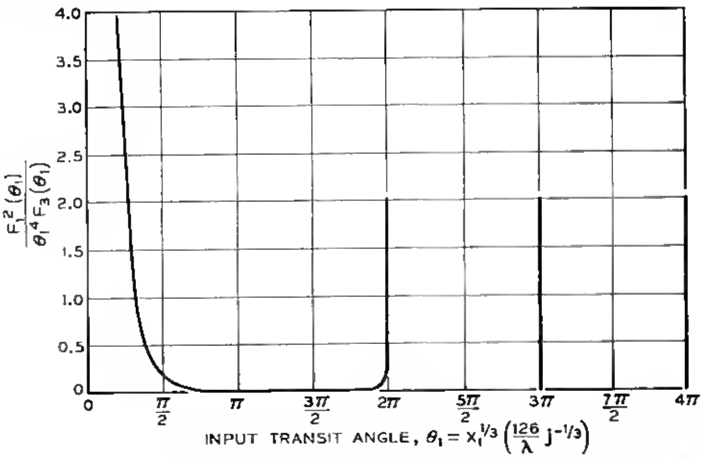

Fig. 7.—Gain-band product dependence on input spacing (x_1), with current density (j) fixed.

Variation with Cathode-Grid Spacing, x_1

Now consider that j has been fixed at the largest permissible value according to the previous section and consider the second formulation

for M_i. The spacing x_1 is involved only in the second factor which again is a function only of $\theta_1 = \left(\frac{x_1}{j}\right)^{1/3}$. We again have a strong first maximum at $\theta \rightarrow 0$ requiring x_1 to be as small as possible (Fig. 7). Other maxima are indicated at larger values of θ_1 (and larger values of x_1) again at points where $G_{11} \rightarrow 0$ and the same remarks apply here as were made in the previous section. For broad-band optima we are therefore interested in minimum values of x_1.

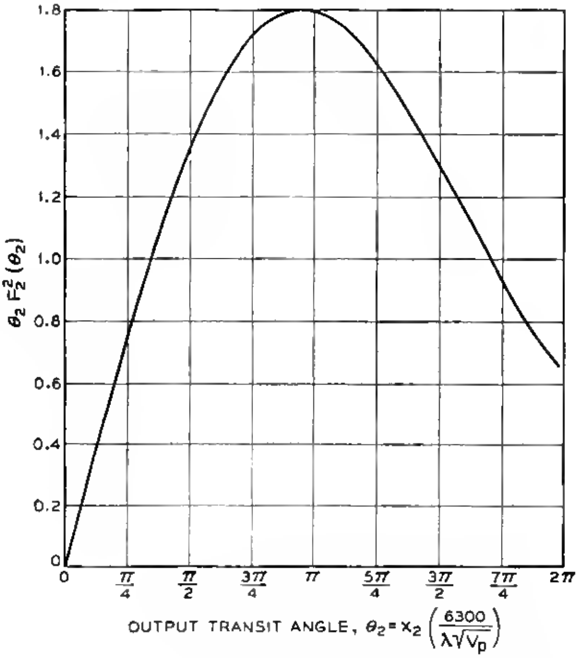

Fig. 8.—Gain-band product dependence on output spacing (x_2).

VARIATION WITH ANODE-GRID SPACING, x_2

The anode-grid spacing x_2 is involved only in the third factor of either formulation. This factor is a function of output transit angle θ_2 and exhibits a maximum for $\theta = 2.9$ radians as shown in Fig. 8. This optimum at a fairly large value of θ_2 is due to the fact that the capacitance C_{22} varies as $1/x_2$ whereas the coupling coefficient of the stream to the gap decreases more slowly at first than the capacitance so that the ratio Y^2_{21}/C_{22} improves as the spacing becomes moderately wide. The optimum θ_2 corresponds to an optimum value of x_2 which of course depends upon the plate voltage and frequency of operation. For the 1553 at 250 volts and 4000 Mc/s, the optimum output spacing is .022″.

Limitations in Choosing Optimum Parameters

Generally, there are mechanical, thermal, emission and specification limits whicb prevent the realization of optimum values for all of the above parameters simultaneously. A good design is one in whicb a nice balance is effected between these various optima and their limitations.

Limitations on Emission Current Density, j

It is generally true that the life of a thermionic electron tube varies inversely as the average cathode current density in a complicated fasbion. The maximum permissible value of j is therefore always a compromise between our desire for highest figure of merit and long life. In the present state of the cathode art as it bas been evolved for tbe 1553 triode it is possible to operate at a current density of 180 ma/cm^2 and obtain an average life of several thousands of hours. It is perhaps of interest to note that it was necessary to develop mucb more dense and smootb oxide coatings in order to make possihle such life in the thin coatings necessary for operation at such close spacings.

Limitations on Cathode-Grid Spacing, x_1

Consider the limitations in reaching the optimum in x_1. There is, of course, the ohvious one that it is mechanically and electrically not possible at present to make x_1 equal to zero and still retain the essential features of unilateral controlled space charge flow. Granting then tbat the spacing cannot be zero, we must choose the smallest value of x_1 for which parallelism and reasonable tolerances can be maintained. To tbis end in the 1553 a value of $x_1 = .0006''$ is very near this limit with present structures.

There is, bowever, at present another limitation wbich is essentially mechanical in nature but makes itself felt electrically in a way not indicated in the ahove simplified theory. This theory has assumed that the grid dimensions are infinitesimally thin compared to the electrode spacings. However, if this is not the case then the grid has less control action than an ideal fine grid, and the intrinsic gain band product must be reduced by still anotber factor F_4 which is a function of the grid transmission factor $a = \frac{p - d}{p}$ and tbe ratio $\frac{x_1}{p}$ wbere p is the pitch distance between grid wire centers and d is the diameter of grid wires. This function has the form shown in Fig. 9.*

Thus if the grid pitch and wire diameter are mecbanically limited to some finite though small values, the optimum in input spacing x_1 will

* Data transmitted informally from C. T. Goddard and G. T. Ford.

still be for $x_1 \rightarrow 0$ but will not increase so strongly as $x^{-4/3}$ as before but much more slowly, about as $x^{-1/3}$. The grid dimensions should consequently be made as small as possible while still maintaining a transmission fraction at no less than 0.5 and at the same time not allowing mean deviations in pitch more than about 15%.

In the 1553 our best grid techniques today have led to a stretched grid (which does not move appreciably during temperature cycling) having a transmission factor of approximately 0.7, a pitch distance of .001″ and a mean deviation in pitch of less than 15%. For such a grid further decreases in input spacing without refining the grid will not pay off very rapidly, since we are on the maximum slope portion of the function F_4.

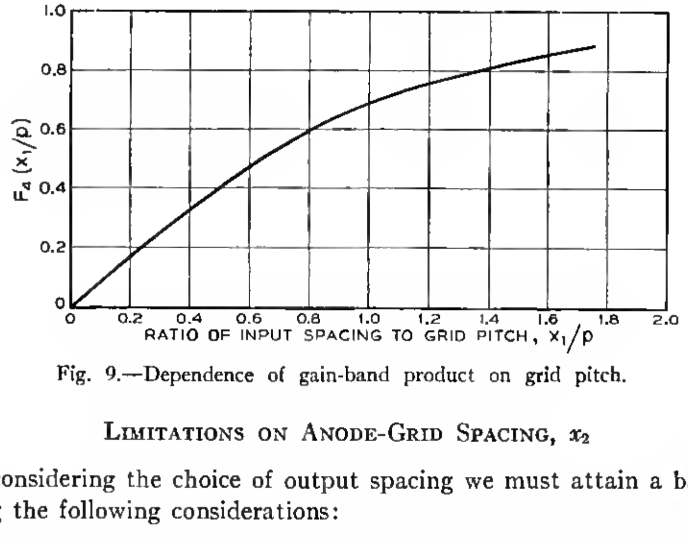

Fig. 9.—Dependence of gain-band product on grid pitch.

Limitations on Anode-Grid Spacing, x_2

In considering the choice of output spacing we must attain a balance among the following considerations:

a. The optimum transit angle $\theta_2 = 2.9$ radians requires a spacing which varies with plate voltage and with frequency. For 250 volts and 4000 Mc/s, this optimum is .022″.
b. The anode heat dissipation must be closely watched because the glass seal in this type of tube is very close to the anode. For the 1553, a maximum of 50 watts per square centimeter of anode active surface is safe. With a maximum cathode current density of 180 ma/cm^2, set by life considerations, heat dissipation limits the plate voltage to 275 volts unless the current is lowered.
c. If the anode is moved too far out, keeping its voltage constant, then in order to draw the desired current the grid must go positive, perhaps drawing excessive grid current. The grid shielding factor μ cannot be reduced without harming the transadmittance and feed-

back values; accordingly the cathode current would have to be reduced below the maximum permissible from life considerations.

d. The circuit degradation factor $(1 + C_{p2}/C_{22})^{-1}$ becomes more unfavorable as the active capacitance C_{22} is reduced by widening the output spacing. For discussion and calculation of this factor, see Appendix 2.

e. A wider output spacing, by virtue of the reduced capacitance, permits a higher maximum frequency limit on the tube.

The actual choice of output spacing in the 1553 is .012″. This compromise between the foregoing factors appears to be suitable at 4000 Mc/s. The output transit angle of 1.6 radians gives 78% of the theoretical optimum intrinsic gain-band product. The anode dissipation is near the maximum safe value for the maximum allowable cathode current. The grid runs very close to cathode potential so that grid current is small. The circuit degradation factor has a value of about 0.8, while the upper frequency limit of the tube is satisfactory (about 5000 Mc/s).

The optimum design just described is an attempt to get the best possible gain-band product in the resulting tube, and is based on a particular electronic theory (that of Llewellyn and Peterson). Two points remain to be discussed. (1) What would be the result of optimizing for other merit figures such as power-band product or noise figure, and (2) how valid is the theory?

Power-Band Product

The radio relay amplifier requires not only gain, but perhaps even more, power output. In such a case, the design specification of greatest importance is the bandwidth over which a certain power output can be obtained with a specified maximum distortion, and is expressed by an analogous figure of merit, the power-band product.

Of the many methods of specifying distortion, one which is particularly useful in this connection is the "compression", that is, the amount by which the gain is reduced from the small-signal value. In an amplitude-modulated system, the compression would be a direct measure of nonlinear amplitude distortion in the amplifiers. In the actual relay, using FM, compression is an indication that the amplifier is approaching its maximum limit of power output.

The maximum power output depends not only on how much current the tube can carry, but also on the magnitude of the load impedance into which this current works, which in turn depends upon the bandwidth of the load. To compare tubes without need of specifying any bandwidth, one notes that the product of power output and band-

width is a constant, a figure of merit. The derivation is outlined in Appendix 1.

$$P_0 \cdot B = \frac{I_{20}^2 F^2(C) F_2^2(\theta_2)}{4\pi C_{\text{out}}} \tag{6}$$

The numerator here is just the square of the maximum ac current; that is, the dc current I_{20}, multiplied by a factor $F(C)$ depending on the allowable compression C, and by the gap coupling coefficient $F_2(\theta_2)$ of the electron stream to the output gap. The latter is of course a function of the output transit angle θ_2. It is assumed that the load is a matched simple resonant circuit and the band is taken 3 db down.

The power optimum must clearly be somewhat different from the gain optimum previously discussed. For example, the transadmittance does not appear here, nor does any property of the input circuit; while the magnitude of the direct electron current, which did not appear in the gain-band product, is now important. The capacitance of the output circuit appears in both figures of merit.

In terms of internal parameters of the tube, application of Llewellyn and Peterson's theory along the lines previously discussed leads to the following expression for power-band product:

$$M_i\,(P) = K[Aj^2\, F^2(C)]\, [\theta_2\, F_2^2\, (\theta_2)\, \sqrt{V_p}] \tag{7}$$

where A is the electrode area, $F^2(C)$ is a function of the allowable distortion limits, K is a constant which may depend upon frequency, and the other symbols are as before.

Considering first the dependence on output transit angle and plate voltage, one sees that this figure of merit has exactly the same form as the gain-band product. It is, however, not quite safe to assume therefore that exactly the same output configuration is still optimum, because the factors entering into the choice of output spacing have not exactly the same relative importance any longer; for example, a positive grid may be less objectionable, or a higher plate voltage may be permissible. Still, as a first approximation one may assume the output configuration to be already somewhere near optimum.

Other factors of the power-band figure of merit show considerable difference from the gain-band product. For instance, the electrode area enters the picture explicitly, suggesting that a larger area tube would give more power. The current density enters squared instead of only to the $\frac{2}{3}$ power; the explicit dependence on input spacing is missing. The compression function $F(C)$ depends mostly on the input conditions in a complicated way difficult to calculate. It can be approximated graphically from static characteristics.

A power tube similar to the 1553 might therefore be larger in electrode area, might have a coarser grid and wider input spacing, and perhaps would differ somewhat in output configuration, particularly if the plate voltage were raised. Any cathode development permitting a higher current density would improve the power output more than the gain, and might well lead to a drastic anode redesign to permit larger plate dissipation.

Similarly, a design to optimize noise figure would lead to still a third version of the tube, in which one might consider such things as critical relationships between input and output spacings.

For the 1553 at 4000 megacycles the following quantitative data may be quoted in order to check the gain-band product estimates.[8]

$$| Y_{21} | = 39.10^{-3} \text{ mhos}$$
$$G_{11} = 73 \cdot 10^{-3} \text{ mhos}$$

Note that the transadmittance is less than the dc value of $45 \cdot 10^{-3}$ mhos by only about 15%, while the input conductance, instead of being equal to the transadmittance as at low frequencies, is almost twice as large, on account of loading of the input gap by electrons returning to the catbode. Using the active capacitance C_{22} of .477 $\mu\mu f$, the intrinsic gain band product is:

$$\Gamma \cdot B = Y_{21}{}^2/4\pi G_{11}C_{22} = 3480 \text{ megacycles.}$$

With the somewhat optimistic capacitance degradation factor of .81 computed in Appendix 2, the gain band product would be reduced to 2820 megacycles.

The experimental average value is about 1100 megacycles.[9] The difference is probably due in part to resistive loss in the passive input circuit, which may be calculated as follows: Neglecting feedback, the input circuit may be represented as containing a resistance R_s in series with the short-circuit input admittance $g_{11} + jb_{11}$. Robertson gives the following values for these elements:

$$g_{11} = 73 \cdot 10^{-3} \text{ mhos}$$
$$b_{11} = 26 \cdot 10^{-3} \text{ mhos}$$
$$R_s = 7.6 \text{ ohms}$$

Accordingly, the input degradation factor $R_{11}/(R_{11} + R_s)$ should be $11.2/(11.2 + 7.6) = .60$, giving a computed overall gain-band product of 1690 megacycles. The best tubes sometimes exceed this figure. Tubes

[8] S. D. Robertson's measurements at 4000 megacycles, *B. S. T. J.*, 28, 619–655 (October 1949).
[9] A. E. Bowen and W. W. Mumford "Microwave Triode as Modulator and Amplifier," this issue of *B. S. T. J.*

with lower values may have excessive input circuit loss or may have narrower bandwidth on the input side than has been assumed. Further measurements, by elucidating this point, might lead to a better design of tube and circuit.

An entirely similar calculation can be made for the power-band product. The additional assumptions required are that the compression function $F^2(C)$ has the conservative value of $\frac{1}{2}$, and the output coupling coefficient $F_2(\theta_2)$ is taken as 0.9. The power-band product at 4000 megacycles is then computed to be 50 watt megacycles, which is quite close to the figures found by Bowen and Mumford.

Refinements of the Electronic Theory

In the electronic computations above, the single-valued theory was used because it is the simplest theory which describes the high frequency case at all accurately. The most important discrepancy between the rigorous theory and the actual situation is the first theoretical assumption listed above, that the electrons are emitted from the cathode with zero velocity. For actual cathodes the velocity of emission is not zero nor uniform but has a Maxwellian distribution such that the average energy away from the cathode is $\frac{1}{2}\,k\,T_k$, or about equivalent to the velocity imparted by a potential drop of 0.04 volt for an oxide cathode at 1000°K. Tbere result several effects whose general nature is known but which have not yet been formulated into a rigorous quantitative theory valid at high frequencies.

(1) A potential minimum is formed at a distance on the order of .001″ in front of the cathode instead of at the cathode as in the simple theory. This distance is not negligihle for close-spaced tubes; so that, for very close spacings, even perfect "physicists' grids" approach a finite trans-conductance limit. [van der Ziel, Philips Research Reports 1, 97–118 (1946); Fig. 2.]

(2) Because the potential minimum implies a retarding field near the cathode many electrons emerging from the cathode are forced to return to it. These returning electrons absorb energy from the signal and also induce excess noise in it, both effects becoming important at high frequencies.

The effects of initial velocities on the figures of merit can be measured experimentally. For example, the circuit and electronic impedances of diodes and triodes at 4000 Mc have been measured by Robertson.[8] Such measurements can determine the electronic loading and noise separately from the circuit degradation effects and are therefore a highly effective

[8] loc. cit.

method of circuit design as well. Robertson found that the input circuit structure of the 1553 produces a measurable impairment in its gain-band product, which redesign of both tube and circuit may be able to improve. Comparison of his results with the theory has given a better understanding of the limits of high-frequency performance, and has lent some support to the following set of rules of thumb which have been in use for some time:

1. The input loading arising from the returning electrons is considerable, the input conductance of these tubes at 4000 Mc being about double the theoretical value of Llewellyn and Peterson.[7]
2. The input noise of these close-spaced tubes checks well with what one would expect of a low-frequency diode with Maxwellian velocities, whose solution is known. In high-frequency noise calculations, therefore, one can use with some confidence Rack's suggestion that cathode noise can be regarded as an effective velocity fluctuation at the virtual cathode.[10]
3. Single velocity theory seems to hold well when velocities are much larger than Maxwellian, drift times are not more than a few cycles, electron beams are short compared to their diameter, and no exact cancellations of large effects are predicted. In particular it holds well for the 1553 output space and for calculations of the high-frequency trans-admittance.

Extensive calculations of signal and noise behavior in planar multigrid tubes have been made by L. C. Peterson, using the single-velocity theory except for an empirical value of input loading, and using Rack's suggestion for cathode noise.[11] The results so far checked have agreed well with experiment.

In short, the optimum design for the tube is still given fairly closely by the figures of merit based on the approximate theory, but the performance will fall somewhat short of the predictions of the simple theory; performance can be estimated with the aid of the experimental measurements and rules of thumb just described.

Summary

From the foregoing calculations we draw a number of conclusions:

1. The figures of merit can be validly analyzed into their dependence on more elementary properties like transadmittance, circuit capacitance, input loss resistance, and so on.

[7] loc. cit.

[10] A. J. Rack "Effects of Space Charge and Transit Time on the Shot Noise in Diodes," *B. S. T. J.*, 17, 592–619 (October 1938).

[11] L. C. Peterson "Space Charge and Noise in Microwave Tetrodes," *Proc. I. R. E.*, 35, 1262–1274 (November 1947).

2. Even rough calculations, such as the coaxial line approximations used in Appendix 2 are close enough to the facts to indicate whether the design is close to an optimum with respect to such parameters as output spacing, anode diameter, grid diameter, and the like. More accurate calculations and experiments can give more precise answers to these questions.
3. Some considerations such as cathode activity, tube life, heater power and so on have not yet been included in the analysis. However, systematic optimization for such parameters as are treated quantitatively is greatly facilitated. In general, each different figure of merit leads to a somewhat different optimum and hence a different version of the tube.

The design of tubes by the method of figure of merit has been outlined. The method is very general, but in essence has just three steps:

1. Formulate the system performance of the projected device with the aid of a figure of merit.
2. Find how the figure of merit depends upon the parameters of the tube, such as spacings, current, etc.
3. Adjust the tube parameters, subject to physical limitations, to optimize the figure of merit.

Acknowledgments

The development of this microwave triode has required not only the expert and highly cooperative services of a large team of electrical, mechanical, and chemical engineers but also the indispensable assistance of skilled technicians, all of whom worked smoothly together to develop these new materials and techniques to a point where they are specifiable and amenable to quantity production. It is not practical to mention all those who have made significant contributions to this development. The contributions of A. J. Chick, R. L. Vance, H. E. Kern and L. J. Speck, however, are of such outstanding nature that mention of them cannot be omitted.

APPENDIX 1

Derivation of the Figures of Merit

Gain-Band Figure of Merit

Let the problem be stated as the design of an amplifier tube to operate with as large gain over as wide a frequency band as practicable. As a standard environment, we use a single-stage amplifier working between equal resistive impedances. For three reasons this standard is suitable: it is simple; it corresponds closely to practicality in many cases especially

in the microwave field; and in most cases, it turns out that performance is limited by the same transadmittance to capacitance ratios as apply when the source and load impedances are not purely resistive. The terminology of high frequencies will be used but the analysis applies at all frequencies under the conditions stated.

Consider the over-all single-stage amplifier of Fig. A1-1 consisting of input resonator, tube and output resonator, to be a single transducer

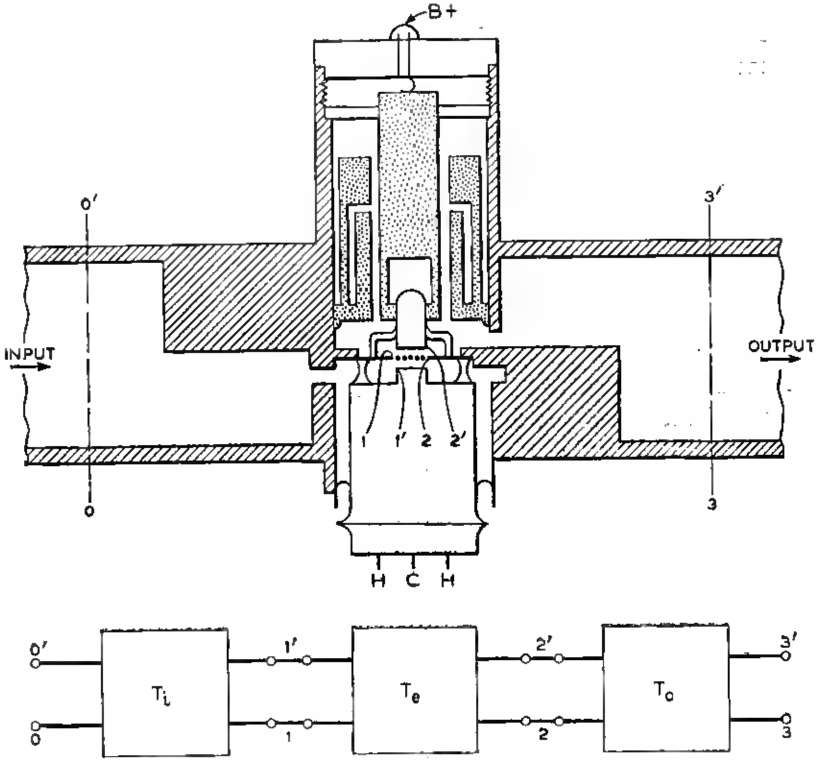

Fig. A1-1.—Microwave triode amplifier.

whose gain and bandwidth we wish to relate to the geometry and other pertinent characteristics of the circuits, bulb and electrode characteristics.

It is instructive to consider the whole transducer to be made up of three transducers in tandem as follows:

1. The input passive transducer, extending from the externally available input terminals (perhaps located somewhere in the driving wave guide or coaxial line) up to the internal input electrodes right at the boundary of the electron stream. Call this transducer T_i ; in the

case of the grid-return triode of Fig. 1 it begins somewhere in the input wave guide at 0-0′ where only the dominant wave exists, includes the input external cavity and that portion of the tube interior right up to but not including the cathode-grid gap adjacent to the electron stream at 1-1′.

2. The output passive transducer, extending from the externally available output terminals located in the output wave guide through the output part of the bulb right up to the internal output electrodes at the boundary of the electron stream. Call this T_0; in the triode it

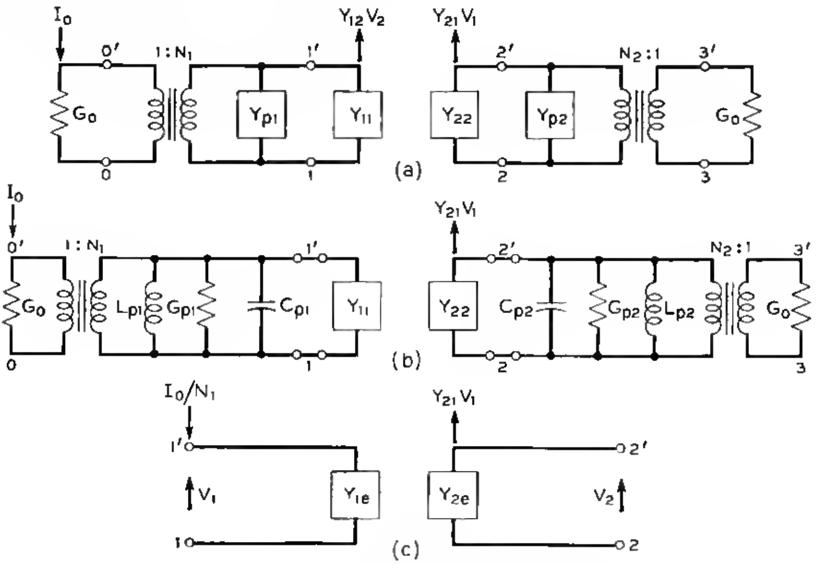

Fig. A1-2.—Amplifier representations.

extends from somewhere in the output wave guide at 3-3′ where only the dominant wave exists, includes the external coupling window, resonator cavity and output portion of the bulb, right up to the grid-anode gap adjacent to the electron stream at 2-2′.

3. The active electron transducer enclosing everything between the internal terminals of the above two passive coupling transducers—call this T_e—in the triode it extends from the cathode-grid gap adjacent to the electron stream at 1-1′ to the grid-anode gap adjacent to the electron stream at 2-2′. Geometrically it includes the stream and active portions of the electrodes. The term "active" will be applied to the electron stream and to those portions of the electrodes which interact directly with the stream.

We may represent these three transducers as in Fig. A1-2a, where the input and output transducers have each been replaced by an ideal transformer of turns ratio N and a shunt admittance Y_p. This representation is general enough for present purposes, provided that Y_p and N are allowed to be complex functions of frequency and provided that terminals 0-0′ and 3-3′ are chosen so that a potential minimum occurs at those points when points 1-1′ and 2-2′ are shorted.

The short-circuit admittances for the whole transducer as seen at terminals 0-0′ and 3-3′ are then

$$\begin{aligned} Y^*_{11} &= N_1^2\,(Y_{11} + Y_{p1}) \\ Y^*_{22} &= N_2^2\,(Y_{22} + Y_{p2}) \\ Y^*_{21} &= N_1 N_2 Y_{21} \\ Y^*_{12} &= N_1 N_2 Y_{12} \end{aligned} \tag{A1-1}$$

where the Y_{ij} are the short-circuit admittances of the electron transducer alone as seen at terminals 1-1′ and 2-2′.

If the feedback admittance Y_{12} is assumed negligible the insertion voltage gain may be written as

$$\Gamma(\omega) = \frac{2N_1 N_2 Y_{21}}{G_0(1 + \sigma_1)(1 + \sigma_2)}$$

where the sigmas are admittance-matching factors:

$$\sigma_1 = \frac{N_1^2(Y_{11} + Y_{p1})}{G_0} = \frac{Y^*_{11}}{G_0}; \qquad \sigma_2 = \frac{N_2^2(Y_{22} + Y_{p2})}{G_0} = \frac{Y_{22}{}^*}{G_0} \tag{A1-2}$$

The gain is maximum when σ_1, σ_2 are minimum, i.e., when tube and circuits are resonant and losses are minimum.

We may rewrite this in terms of the total Y^*_{ij} as follows:

$$\Gamma(\omega) = \frac{2Y^*_{21}}{G_0(1 + \sigma_1)(1 + \sigma_2)} \tag{A1-3}$$

Many practical cases are well approximated by the more special representation of Fig. A1–2b, where the turns ratios of the ideal transformers are real and independent of frequency, and the shunt admittance consists of ordinary lumped constant circuit elements. The feedback admittance Y_{12} is neglected.

This representation as simple, lumped-constant elements holds very well for any admittance, even a distributed, cavity-type microwave circuit, or an electronic admittance, provided that the combined circuit has no series and only one shunt resonance near the frequency band in

question. The "effective values" of the actual admittance are given by equations (3) of the text, as follows:

$$\begin{aligned} G_p &= G_x\,(\omega_0) \\ C_p &= \tfrac{1}{2}\,(B_x' + B_x/\omega_0) \\ \frac{1}{L_p} &= \tfrac{1}{2}\,(\omega_0^2\,B_x' - \omega_0\,B_x) \end{aligned} \tag{A1-4}$$

Let the complete admittances across nodal pairs 1-1′ and 2-2′ be called Y_{1e} and Y_{2e} as in Fig. A1-2c, which is an abbreviation of Fig. A1-2b from the point of view of the active transducer.

$$\begin{aligned} Y_{1e} &= G_1 + G_{p1} + G_{11} + j\omega C_{p1} + j\omega C_{11} + \frac{1}{j\omega L_{p1}} \\ &= G_{1e} + j\omega C_{1e} + \frac{1}{j\omega L_{1e}} \\ Y_{2e} &= G_2 + G_{p2} + G_{22} + j\omega C_{p2} + j\omega C_{22} + \frac{1}{j\omega L_{p2}} \\ &= G_{2e} + j\omega C_{2e} + \frac{1}{j\omega L_{2e}} \end{aligned} \tag{A1-5}$$

where G_1 and G_2 are the line admittances as seen from the active transducer:

$$G_1 = G_0/N_1^2;\; G_2 = G_0/N_2^2.$$

The Q's of the circuit are defined as

$$\begin{aligned} Q_{1e} &= \omega_0\,C_{1e}/G_{1e} \\ Q_{2e} &= \omega_0\,C_{2e}/G_{2e} \end{aligned} \tag{A1-6}$$

The insertion voltage gain (2) may be written as follows to emphasize the manner in which it depends upon frequency:

$$\Gamma = \frac{2Y_{21}}{Y_{1e}Y_{2e}}\sqrt{\frac{G_{1e}G_{2e}}{(1+\mu_1)(1+\mu_2)}} \tag{A1-7}$$

Here $\mu = \sigma(\omega_0)$ is the matching factor at band center. Frequently the circuits are matched ($\mu_1 = \mu_2 = 1$) to avoid standing waves in system applications, and we shall discuss this case; but in any case μ_1 and μ_2 are constants with respect to frequency. For our standard circuits, G_{1e} and G_{2e} are independent of frequency; also ordinarily the transadmittance Y_{21} may be considered constant for bandwidths commonly encountered. There results then the fact that the voltage gain (and phase) depends on frequency in the same way as $(Y_{1e}\,Y_{2e})^{-1}$.

Since the gain varies with frequency, the amplifier will give approximately constant response only within a certain range of frequencies. The band of the amplifier is defined as that frequency interval within which the magnitude of the gain is constant within some specified tolerance; the bandwidth is the size of this interval. We wish to express the gain of the amplifier in terms of its bandwidth, in the following way:

The voltage gain of this amplifier has a maximum, called Γ_0, at band center frequency f_0. Take the band of the amplifier $B_N(A)$ as that interval within which the voltage gain is within a factor of 1/N times the maximum.

$$\left| \frac{\Gamma(\omega)}{\Gamma(\omega_0)} \right| \geq \frac{1}{N} \text{ defines } B_N(A) \tag{A1-9}$$

We can analogously define the band of a simple circuit $B_n(C)$ by the relation

$$\left| \frac{Y_{2e}(\omega_0)}{Y_{2e}(\omega)} \right| \geq \frac{1}{n} \text{ defines } B_n(C). \tag{A1-9}$$

It follows directly that

$$B_n(C) = \frac{G_{2e}}{2\pi C_{2e}} \sqrt{n^2 - 1}\,. \tag{A1-10}$$

Since the amplifier gain is inversely proportional to the product of the circuit admittances, it follows that $n_1 n_2 = N$.

The intrinsic bandwidth resulting from the tube admittance may not be suitable for the intended application. In that case the band may he widened by increasing G_{1e} or G_{2e} with a corresponding decrease in gain. We have then the problem of adjusting G_{1e} and G_{2e} for greatest band efficiency, i.e., maximum gain for a given bandwidth, with synchronous tuning. It turns out that if the bandwidth is less than that needed, then the circuit of higher Q should be lowered until either (a) the band becomes wide enough, or (b) the Q's become equal. In case (b), both Q's should then be lowered, maintaining equality, until the band is wide enough.

Two important limiting cases are to he considered: (a) $Q_{1e} = Q_{2e}$, i.e. the band is shaped equally by the input and output circuits; and (b) $Q_{1e} << Q_{2e}$, i.e. the band is shaped by only the output circuit. In the equal-Q case we have

$$\frac{G_{1e}}{C_{1e}} = \frac{G_{2e}}{C_{2e}}$$

$$n^2 = N \tag{A1-11}$$

$$B_N(A) = \frac{1}{2\pi} \sqrt{\frac{G_{1e}}{C_{1e}} \frac{G_{2e}}{C_{2e}}} \sqrt{N - 1}\,.$$

If only the output circuit is involved, then $N = n_2$ and the band of the amplifier, heing shaped differently, is given hy a different relation:

$$B_N(A) = \frac{G_{2e}}{2\pi C_{2e}} \sqrt{N^2 - 1}. \tag{A1-12}$$

In other words, a hand shaped by only one circuit has the shape of (12), while a band shaped hy two circuits has the shape (11). The maximum voltage gain is

$$| \Gamma_0 | = | \Gamma(\omega_0) | = \frac{2 | Y_{21} |}{\sqrt{G_{1e}G_{2e}(1 + \mu_1)(1 + \mu_2)}} \tag{A1-13}$$

Suhstituting for the G's in terms of the bandwidth, we have for the equal-Q case (from 11)

$$| \Gamma_0 | = \frac{| Y_{21} |}{2\pi\sqrt{C_{1e}C_{2e}}} \frac{2\sqrt{N - 1}}{\sqrt{(1 + \mu_1)(1 + \mu_2)}} \frac{1}{B_N} \tag{A1-14}$$

and for the unequal-Q case (from 12)

$$| \Gamma_0 | = \frac{| Y_{21} |}{\sqrt{G_{1e}} \sqrt{4\pi C_{2e}}} \frac{\sqrt{8} \sqrt[4]{N^2 - 1}}{\sqrt{(1 + \mu_1)(1 + \mu_2)}} \frac{1}{\sqrt{B_N}} \tag{A1-15}$$

These equations give the relationship between the gain and bandwidth of a transmission system shaped hy two or one independent circuits, respectively. The comparison hetween these two cases is not quite straightforward. First, the hand shapes (11) and (12) are different, although this difference is small enough to he ignored for $N < 2$ (6 db down). Second, the gain varies differently as the hand is widened; the equal-Q case loses gain at 6 dh per octave in handwidth, the unequal-Q case only 3 db per octave. The comparison therefore depends on the bandwidth chosen. However, these formulas are still quite useful, especially in comparing two amplifiers of the same type or in optimizing an amplifier of one of the types.

From the equal-Q formula one notices that the product of insertion voltage vain and bandwidth does not depend on the bandwidth, but is a figure of merit by which two amplifiers of the same type (i.e. equal Q) but different gains and bandwidths can he compared. Since

$$C_{1e} = C_{11} + C_{p1} ; C_{2e} = C_{22} + C_{p2}$$

$$| \Gamma_0 | \; B_N = \left(\frac{Y_{21}}{2\pi\sqrt{C_{11}C_{22}}}\right)\left(\frac{1}{\sqrt[4]{1 + \frac{C_{p1}}{C_{11}}} \sqrt[4]{1 + \frac{C_{p2}}{C_{22}}}}\right) \cdot \left(\frac{2\sqrt{N - 1}}{\sqrt{(1 + \mu_1)(1 + \mu_2)}}\right) \tag{A1-16}$$

This expression for the gain-band figure of merit of a two-circuit, line-to-line amplifier is particularly useful for grounded-cathode pentodes and klystrons. It is the product of three factors. The first may be called the electronic figure of merit because it depends only upon electron stream parameters (ratio of transadmittance to mean capacitance of the electronic transducer T_e). The second is the degradation factor giving the effect of adding passive circuit capacitance both inside and outside the bulb to the active capacitance already present in the electronic transducer. The third factor, called the matching factor, depends only on the matching conditions and on the arbitrary definition of bandwidth. If the band is taken 6 db down (3 db for eacy circuit) and the tube input and output are matched, the third factor is unity.

In amplifiers using triodes and tetrodes in grid-return circuits, the Q of the input circuit is usually very much smaller than that of the output. Here it is appropriate to use tbe single-circuit limiting concept, with $Q_{1e} << Q_{2e}$. Here a figure of merit independent of bandwidth is obtained from the product of power gain and bandwidtb:

$$|\Gamma_0|^2 \, B_N = \left(\frac{|Y_{21}|^2}{4\pi G_{11} C_{22}}\right)\left(\frac{1}{\left(1 + \frac{G_{p1}}{G_{11}}\right)\left(1 + \frac{C_{p2}}{C_{22}}\right)}\right) \left(\frac{8\mu_1\sqrt{N^2 - 1}}{(1 + \mu_1)^2(1 + \mu_2)}\right) \qquad \text{(A1-17)}$$

This expression for the gain-band figure of merit of a one-circuit, line-to-line amplifier is also the product of three factors. The first is again the intrinsic electronic figure of merit of the active transducer alone; the second is the degradation produced by the addition of passive circuit capacitance to the output and circuit loss to the input; the third is a band-definition matching factor which is unity when the band is taken 3 db down and the tube is matched.

In the application of the figures of merit, the third factors are usually omitted, since they depend only on the matching conditions and on the particular definitions of bandwidth used.

Power-Band Figure of Merit

In the problem of power output amplifier stages, the design specification of greatest importance is the bandwidth over which a certain power output can be obtained with a specified maximum of distortion. Of the many methods of specifying distortion, one which is particularly useful for microwave systems is known as the "compression". If the power gain is plotted in decibels as a function of the power output, as shown in Fig.

A1-3, it will normally be constant for low power levels (for which the device is essentially linear) and equal to the low level power gain $|\Gamma|^2$. However, at some higher power level non-linearities appear in some or all of the various short-circuit admittances, usually causing the power gain to decrease below the small-signal value by an amount called the compression, C. If P_0/P_i be power gain for any power output and $|\Gamma|^2$

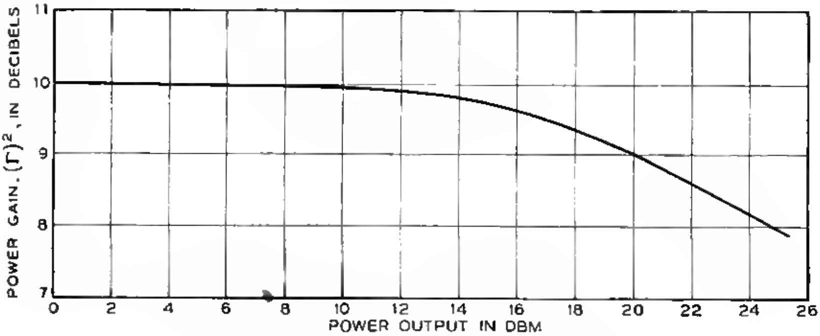

Fig. A1-3.—Typical gain variation with power output.

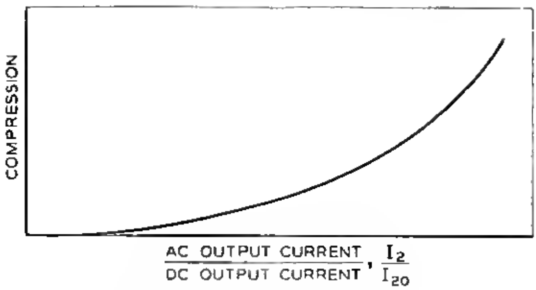

Fig. A1-4.—Compression vs. $\dfrac{\text{Alternating current in output}}{\text{Direct current in output}}$

the small-signal power gain, the compression C is defined in decibels as follows:

$$C = 10 \log_{10} |\Gamma|^2 - 10 \log_{10} P_0/P_i \tag{A1-18}$$

$$= 10 \log_{10} \frac{|\Gamma|^2 P_i}{P_0}$$

Naturally, the compression depends upon how hard the tube is driven. It is therefore a function of the amount of drive, which may be conveniently expressed in terms of the ratio of the alternating output current to the operating direct current, as in Fig. A1-4.

The power output depends on operating parameters thus:

$$P_0 = I_2^2 \frac{G_2}{(G_{22} + G_2)^2} \tag{A1-19}$$

As the output power level is continually raised, more and more current is required to drive the load, until finally the non-linear distortion limit is reached. The maximum output current is therefore limited to a certain proportion of the direct current I_{20}, thus:

$$I_{2m} = I_{20} \cdot F(C)\ F_2(\theta_2) \tag{A1-20}$$

where $F(C)$ shows the dependence upon the compression C and will naturally be the larger, the more the allowable compression. $F_2(\theta_2)$ indicates a dependence upon output transit angle; it is the output gap coupling coefficient.

The power output depends also upon the output circuit conductance G_2 and can be greater if G_2 is smaller. However, a smaller G_2 implies a smaller bandwidth. It results that the power is inversely proportional to the bandwidth of the output circuit, or in other words, the product of power output by the bandwidth of the output circuit is a constant—a figure of merit of the tube. As in the case of the gain-band merit, this also can be broken up into factors:

$$P_0 \cdot B_N = \left(\frac{I_{20}^2 F^2(C) F_2^2(\theta_2)}{4\pi C_{22}}\right)\left(\frac{1}{1 + C_{p2}/C_{22}}\right)\left(\frac{2\sqrt{N^2 - 1}}{1 + \mu_2}\right) \tag{A1-21}$$

This expression for the power-band figure of merit is the product of three factors. The first is the intrinsic figure of merit of the active transducer alone; the second is the degradation caused by the addition of passive circuit capacitance to the output circuit; the third is a band definition—matching factor which is unity when the output is matched and the band of the output circuit is taken 3 db down.

The power-band computation does not depend upon the input circuit. Variations in the latter affect the gain of the amplifier, but not its overload point. Accordingly in the power band formula only properties of the tube and its output circuit appear. When feedback has to be considered, then the input circuit also affects the power, and the analysis becomes more complicated.

We have now three figures of merit: namely, two gain-band products applying to different kinds of amplifiers, and one power-band product. They relate the performance of an amplifier to certain internal parameters. For wide band service, the tube design should make the appropriate figure of merit as large as practicable.

It should be understood that many other factors may have a bearing on amplifier design, such as power consumption, noise performance or amount of feedback. Where such factors are important, they too must be considered, and frequently appropriate merit figures like plate efficiency or noise figure are useful.

APPENDIX 2

The Circuit Capacitance Degradation Factor

The capacitance degradation factor $C_{22}/(C_{22} + C_{p2})$ which applies to both gain-band and power-band products, can be calculated approximately

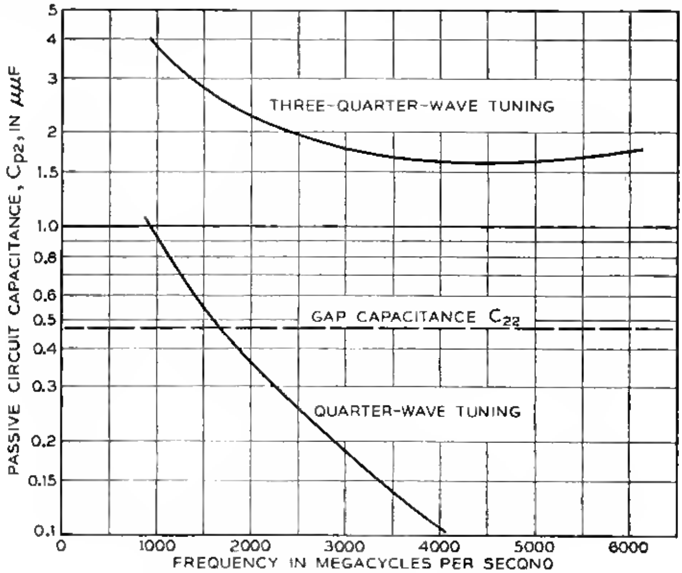

Fig. A2-1.—Passive circuit capacitance C_{p2}.

as shown below. As the frequency is varied, this factor changes by considerable amounts for the 1553 tube; accordingly, both figures of merit vary with frequency, and design control has been exercised to produce maximum merit around 4000 megacycles.

The capacitance degradation factor is just the proportion which the active tube capacitance bears to the total capacitance of tube and circuit, and would therefore have a maximum of unity if the circuit passive capacitance were made zero. For the 1553, we may begin by assuming that the plate circuit is to be tuned by a resonant coaxial line. As the frequency is lowered the effective capacitance will be increased, since the line must be lengthened; its variation is shown in Fig. A2-1.

The calculation is based on the following assumptions (Fig. A2-2):

1. The output cavity has inner diameter .180″, outer .850″, consequently a characteristic admittance G_0 :

$$G_0 = 7250/\log \frac{d_2}{d_1} = 10{,}710 \text{ micromhos} \tag{A2-1}$$

2. The gap capacitance is that of a parallel plate condenser of .180″ diameter and .012″ spacing, namely

$$C_{22} = \epsilon_0\, A/d = 0.477\ \mu\mu f \tag{A2-2}$$

3. The effect of the glass vacuum envelope is neglected for simplicity.

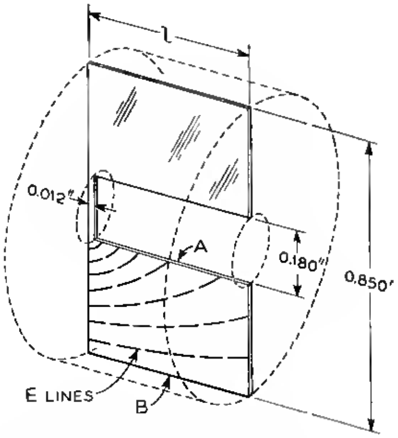

Fig. A2-2.—Output cavity dimensions. A, B are concentric cylindrical portions. Actual lines of electric force are partly dotted into sketch.

Consequently the length l of the line is given by the well-known tuning relation

$$\omega C_{22} = G_0 \cot \theta = G_0 \cot \frac{\omega l}{c} \tag{A2-3}$$

The distributed capacitance of the line is determined from the formulas (3) of the text, which in this case reduces to the following:

$$\omega C_{p2} = \frac{G_0\theta}{2}\left(1 + \frac{\omega^2 C_{22}^2}{G_0^2}\right) - \frac{\omega C_{22}}{2} \tag{A2-4}$$

The cavity distributed capacitance is thus comparatively easy to calculate at high frequencies because of the simplicity of the geometry. At low frequencies the computation of the distributed capacity of a coil is no

different in principle, but would be harder to carry out in practice because of the helical geometry. The value can of course in any case be found by measurement of the tuning admittance as a function of frequency. From these equations the circuit degradation factor can be calculated, and is shown in Fig. A2-3 as a function of frequency.

The accuracy of the coaxial line assumptions decreases as the cavity becomes shorter. For 4000 and 6000 megacycles, since the length of the cavity is less than its diameter, it would be more nearly correct to regard it as a radial transmission line loaded by the inductive "nose" in the

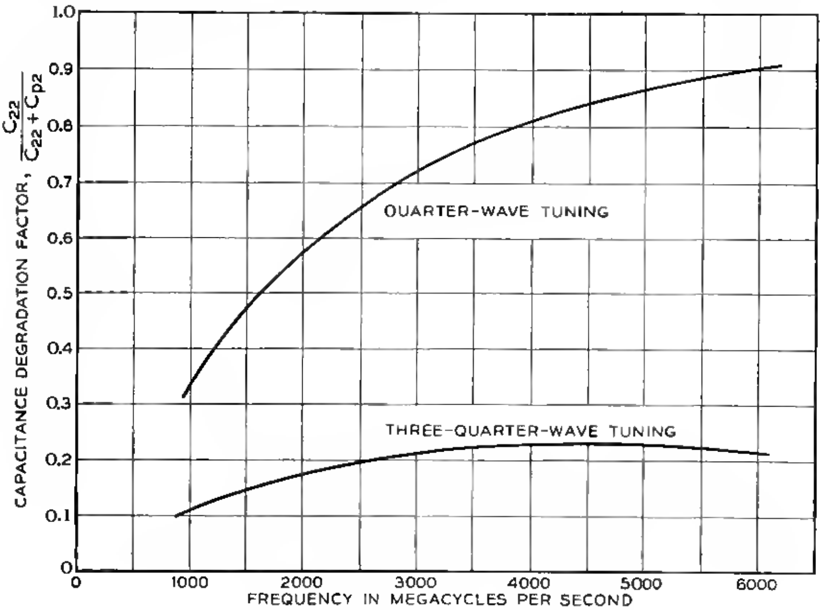

Fig. A2-3.—Capacitance degradation factor, $\frac{C_{22} + C_{p2}}{C_{22}}$.

center. The admittance of such a cavity can be calculated[12] or measured; but the additional precision hardly warrants the effort in the present case.

The capacitance degradation factor at 4000 megacycles is indicated from Fig. A2-3 as .81, or only 0.9 db less than the intrinsic limit of unity if the passive capacitance were entirely negligible compared to the active 0.5 $\mu\mu f$. This indication is somewhat optimistic, as appears from Fig. A2-2. The coaxial line formulas assume that the capacitance corresponds to a radial electric field between concentric cylinders A and B. This capacitance is found to be quite small (.11 $\mu\mu f$ at 4000 Mc.). The actual lines of

[12] S. Ramo and J. R. Whinnery, "Fields and Waves in Modern Radio," N. Y., Wiley, 1944.

force, dotted in the figure, clearly correspond to a somewhat larger capacitance, especially when the length of the cavity is smaller than its diameter; but this larger capacitance is probably still less than the active capacitance C_{22}.

In so far as the gain-band product depends on the circuit capacitance degradation factor (Fig. A2-3), the curve is probably fairly accurate up to 2000 megacycles and somewhat optimistic for higher frequencies where the coaxial line predictions are evidently too small.

Above 5000 megacycles the quarter-wave tuning cannot be used for the 1553 tube since the glass would interfere with the tuning plunger. A glance at Fig. A2-3 shows that moving the plunger back a half-wave to the next node involves a drastic loss in gain-band product—a factor of four at 6000 megacycles—because of the great increase in circuit passive capacitance. Redesign of the tube for good figure of merit at 6000 megacycles would therefore require the use of first-node tuning. A reduction in outer diameter would be necessary, and the use of an internal pre-tuned cavity might also be indicated.